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The influences of surface polarization on NLC cells

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The influence of the surface polarization, $\mathbf{P_s}$, on a nematic liquid crystal (NLC) cell is investigated analytically. Flexoelectric polarization is considered, but selective ion absorption is ignored. The differential equations are derived for tilt angle, θ, of director **n** and the corresponding boundary conditions based on Gibbs free energy, and their solutions discussed. Equations for the reduced threshold voltage, u_{th}, and the reduced saturation voltage, u_{sat}, are deduced and the relationships between u_{th}, u_{sat} and reduced strength of surface polarization, p, derived.

1. Introduction

Prost and Marcerou [1] first propounded that variance of the nematic order parameter, S, along with position results in electric polarization. Barbero *et al.* [2] investigated this subject again and derived the formula:

$$\mathbf{P}_s = r_1(\mathbf{n}\cdot\mathrm{grad}S)\mathbf{n} + r_2(\mathrm{grad}S), \qquad (1)$$

where r_1 and r_2 are order electric coefficients and **n** is the director of a nematic liquid crystal (NLC). $\mathbf{P_s}$, which only exists in an LC surface layer with thickness 10 nm, is called as the surface polarization.

Surface polarization is present if **n** is fixed and S is changed along with position. On the other hand, when S is fixed and **n** is changed there will be another polarization. This is the flexoelectric polarization [3], denoted as $\mathbf{P_f}$, which can be expressed as:

$$\mathbf{P}_f = e_1\mathbf{n}(\nabla\cdot\mathbf{n}) + e_3(\nabla\times\mathbf{n})\times\mathbf{n}. \qquad (2)$$

Equation (2) consists of two terms with corresponding flexoelectric coefficients, e_1 and e_3, related to the splay and bend distortions, respectively. In an LC cell, **n** has to vary along with position, so $\mathbf{P_f}$ exists in whole LC volume.

Up to now the effects and influences of flexoelectric polarization have been investigated comparatively sufficiently [4], but few articles have been reported about quantitative investigation of surface polarization. Recently, Zakharov and Dong [5] investigated the relation between surface polarization, $\mathbf{P_s}$, and effective anchoring energy. They supposed that when the solid substrate of an NLC cell is in contact with nematic, selective ion adsorption takes place, and the surface electric field originated from this surface charge may be coupled with surface polarization to form interface energy to obtain the expression for effective anchoring energy. The influence of surface polarization on NLC cell is reflected by effective anchoring energy. However, in their derivation, the interface energy induced by surface polarization has to depend on selective ion adsorption. If there is little or no ion adsorption, then this interface energy does not exist or can be ignored. Nazarenko and Lavrentovich [6] reported that the density of adsorptive ions is determined by cleaning techniques and time. They found that cleaning for a long time eliminated adsorptive ions. However, the effect of flexoelectric polarization was not considered [6]. Subsequently, Blinov *et al.* [7] have measured the values of surface polarization and flexoelectric polarization in an NLC cell; they found that these two values had the same order of magnitude.

In this paper, we also investigate the influence of surface polarization on an NLC cell. We consider that there is also flexoelectric polarization but assume that ion adsorption can be ignored. The expression for Gibbs free energy, G, is derived and the differential equation and boundary conditions deduced of the tilt angle, θ, of director **n** by varying G under fixed voltage, U. The solution of director distribution can be obtained by means of numerical calculation. As usual, the two parameters of threshold voltage and saturation voltage are used to describe the characteristic properties of the NLC cell. Equations satisfied by these two parameters are given; hence the influence of surface polarization on an NLC cell is obtained.

The paper is arranged as follows. In section 2, we derive the Gibbs free energy, G, of a planar alignment NLC cell, and deduce the differential equation for tilt angle of

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director **n** and corresponding boundary conditions. In section 3, we discuss the solutions of this differential equation under boundary conditions. The equations satisfied by threshold voltage and saturation voltage are derived in section 4. From the curves of reduced threshold voltage versus reduced strength of surface polarization and reduced saturation voltage versus reduced strength of surface polarization, the influence of surface polarization can be seen. The results are discussed in section 5.

2. The Gibbs energy and the fundamental equations

Consider a planar alignment NLC cell. Taking the z-axis as the direction perpendicular to the cell surface (two substrates lie in $z=0$ and $z=l$ directions), the x-axis parallel to the easy direction of anchoring, θ the angle between the director **n** and x-axis, the surface polarization, $\mathbf{P_s}$, and flexoelectric polarization, $\mathbf{P_f}$, are

$$\mathbf{P_s}=\sin\theta\cos\theta\frac{\mathrm{d}S}{\mathrm{d}z}\mathbf{i}+\left(r_1\sin^2\theta+r_2\right)\frac{\mathrm{d}S}{\mathrm{d}z}\mathbf{k}, \tag{3}$$

$$\mathbf{P_f}=\left(e_1\cos^2\theta+e_3\sin^2\theta\right)\frac{\mathrm{d}\theta}{\mathrm{d}z}\mathbf{i}+\left(\frac{e_1+e_3}{2}\sin 2\theta\right)\frac{\mathrm{d}\theta}{\mathrm{d}z}\mathbf{k}. \tag{4}$$

When the applied field is $\mathbf{E}=(0,\ 0,\ E_3)$, the electric displacement, $\mathbf{D}$, can be expressed as:

$$\mathbf{D}=\varepsilon_\perp\mathbf{E}+\Delta\varepsilon(\mathbf{E}\cdot\mathbf{n})\mathbf{n}+\mathbf{P_f}+\mathbf{P_s}, \tag{5}$$

and its third component is a constant and can be expressed as:

$$D_3=\left(\varepsilon_\perp+\Delta\varepsilon\sin^2\theta\right)E_3+P_{f3}+P_{s3}, \tag{6}$$

where $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the dielectric constants parallel and perpendicular to the director **n**, and $\Delta\varepsilon=\varepsilon_\parallel-\varepsilon_\perp$ is the dielectric anisotropy of the NLC. In general, $\Delta\varepsilon\neq 0$. The voltage, U, between two substrates can be expressed as

$$U=\int_0^l E_3\,\mathrm{d}z=\int_0^l\frac{D_3}{\left(\varepsilon_\perp+\Delta\varepsilon\sin^2\theta\right)}\mathrm{d}z-\int_0^l\frac{P_{f3}+P_{s3}}{\left(\varepsilon_\perp+\Delta\varepsilon\sin^2\theta\right)}\mathrm{d}z. \tag{7}$$

Let

$$g(\theta)=\left(\varepsilon_\perp+\Delta\varepsilon\sin^2\theta\right). \tag{8}$$

Then

$$D_3=\frac{1}{\int_0^l\frac{1}{g(\theta)}\mathrm{d}z}\left\{U+\int_0^l\frac{P_{f3}+P_{s3}}{g(\theta)}\mathrm{d}z\right\}. \tag{9}$$

Now we consider the Gibbs free energy per unit area of substrate, G. It can be expressed as:

$$G=\int_0^l\mathrm{d}z\left(f_{elastic}+f_{dielec}+f_{polarize}\right)+\left(g_s|_{z=0}+g_s|_{z=l}\right), \tag{10}$$

where $f_{elastic}$, f_{dielec} and $f_{polarize}$ are the Frank elastic, dielectric and polarization free energy densities, respectively, and can be expressed as:

$$f_{elastic}=\frac{1}{2}\left(K_{11}\cos^2\theta+K_{33}\sin^2\theta\right)\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^2, \tag{11}$$

$$f_{dielec}=-\frac{1}{2}\mathbf{D}\cdot\mathbf{E}=-\frac{1}{2}\left(\varepsilon_\perp+\Delta\varepsilon\sin^2\theta\right)E_3^2=-\frac{1}{2}g(\theta)E_3^2, \tag{12}$$

$$f_{polarize}=-\left(P_{f3}+P_{s3}\right)E_3=-\frac{1}{2}(e_1+e_3)\sin 2\theta\frac{\mathrm{d}\theta}{\mathrm{d}z}E_3-\left(r_1\sin^2\theta+r_2\right)\frac{\mathrm{d}S}{\mathrm{d}z}E_3, \tag{13}$$

where $E_3=\frac{D_3-P_{f3}-P_{s3}}{g(\theta)}$ and K_{11} and K_{33} are the Frank splay and bend elastic constants, respectively. The surface anchoring energies, g_s, in equation (10) are:

$$g_s|_{z=0}=\frac{A}{2}sin^2\theta_0, \tag{14}$$

$$g_s|_{z=l}=\frac{A}{2}sin^2\theta_l, \tag{15}$$

where A is the anchoring strength and θ_0 and θ_l are the values of θ at $z=0$ and $z=l$ respectively. Substituting equations (11)–(15) into equation (10), and considering that

$$\frac{\mathrm{d}S}{\mathrm{d}z}\begin{cases}=0 & \delta_1<z<l-\delta_2\\ \neq 0 & 0<z<\delta_1,\ l-\delta_2<z<l\end{cases}, \tag{16}$$

where δ_1 and δ_2 are the thickness of layer in which $\frac{\mathrm{d}S}{\mathrm{d}z}\neq 0$, we obtain

$$G=\int_0^l\mathrm{d}z\frac{1}{2}\left[\left(K_{11}cos^2\theta+K_{33}sin^2\theta\right)+\frac{(e_1+e_3)^2}{4g(\theta)}\sin^2 2\theta\right]\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^2-\frac{1}{2}D_3^2\int_0^l\frac{1}{g(\theta)}\mathrm{d}z-\frac{1}{2}\int_0^{\delta_1}\frac{1}{g(\theta)}P_{s3}\left(P_{s3}+2P_{f3}\right)\mathrm{d}z-\frac{1}{2}\int_{l-\delta_2}^{l}\frac{1}{g(\theta)}P_{s3}\left(P_{s3}+2P_{f3}\right)\mathrm{d}z+\frac{A}{2}\sin^2\theta_0+\frac{A}{2}\sin^2\theta_l. \tag{17}$$

By using equations (17) and (9), we calculate the variation of G by means of a literature method [8] to obtain the differential equation of θ and corresponding boundary conditions as

$$\frac{1}{2}\frac{\partial f_{eff}(\theta)}{\partial\theta}\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^2-\frac{\mathrm{d}}{\mathrm{d}z}\left(f_{eff}(\theta)\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)-\frac{1}{2}D_3^2\frac{\Delta\varepsilon\sin 2\theta}{g(\theta)}=0, \tag{18}$$

$$f_{eff}(\theta_0)\frac{\mathrm{d}\theta}{\mathrm{d}z}\bigg|_{z=0}=\frac{1}{2}\left\{A+\frac{e_1+e_3}{g(\theta_0)}\left(D_3-P_{s3}|_{z=0}\right)\right\}\sin 2\theta_0, \tag{19}$$

$$-f_{eff}(\theta_l)\frac{\mathrm{d}\theta}{\mathrm{d}z}\bigg|_{z=l}=\frac{1}{2}\left\{A-\frac{e_1+e_3}{g(\theta_l)}\left(D_3-P_{s3}|_{z=l}\right)\right\}\sin 2\theta_l, \tag{20}$$

where—

$$f(\theta)=k_{11}\cos^2\theta+k_{33}\sin^2\theta+\frac{(e_1+e_3)^2}{4g(\theta)}\sin^2 2\theta. \tag{21}$$

To give the above equations, we make an approximation by considering $\delta_1\ll l$, $\delta_2\ll l$ and that the values of $\mathbf{P_s}$ and $\mathbf{P_f}$ have the same order of magnitude.

3. The solutions of the fundamental equations

Now we discuss the solutions of equation (18)–(20). If $\frac{\mathrm{d}\theta}{\mathrm{d}z}\neq 0$, then equation (18) yields

$$f(\theta)\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^2-\frac{D_3^2}{g(\theta)}=C, \tag{22}$$

where C is constant. Supposing θ reaches the maximum value θ_m at $z=d$, then

$$C=-\frac{D_3^2}{g(\theta_m)}. \tag{23}$$

Equation (22), by using equation (23), yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}z}=\pm D_3\sqrt{\frac{\Delta\varepsilon\left(\sin^2\theta_m-\sin^2\theta\right)}{f(\theta)g(\theta)g(\theta_m)}}. \tag{24}$$

Let

$$\frac{1}{M(\theta)}=\sqrt{\frac{\Delta\varepsilon\left(\sin^2\theta_m-\sin^2\theta\right)}{f_{eff}(\theta)g(\theta)g(\theta_m)}}. \tag{25}$$

Then

$$\frac{\mathrm{d}\theta}{\mathrm{d}z}=\begin{cases}\dfrac{D_3}{M(\theta)} & 0<z<d\\ -\dfrac{D_3}{M(\theta)} & d<z<l\end{cases}, \tag{26}$$

the solution of which can be expressed as

$$z=\frac{1}{D_3}\int_{\theta_0}^{\theta}M(\theta)\mathrm{d}\theta\quad\text{for}\quad 0<z<d \tag{27}$$

and

$$z=l+\frac{1}{D_3}\int_{\theta}^{\theta_l}M(\theta)\mathrm{d}\theta\quad\text{for}\quad d<z<l. \tag{28}$$

There are five unknown quantities in equations (27)–(28), θ_0, θ_l, θ_m, d and D_3. They should be determined by the thickness of cell, l, the voltage U and the boundary conditions in equations (19)–(20). These quantities may be calculated by following equations:

$$l=\frac{1}{D_3}\int_{\theta_0}^{\theta_m}M(\theta)\mathrm{d}\theta+\frac{1}{D_3}\int_{\theta_l}^{\theta_m}M(\theta)\mathrm{d}\theta \tag{29}$$

or

$$D_3=\frac{1}{l}\left[\int_{\theta_0}^{\theta_m}M(\theta)\mathrm{d}\theta+\int_{\theta_l}^{\theta_m}M(\theta)\mathrm{d}\theta\right]; \tag{30}$$

$$d=\frac{1}{D_3}\int_{\theta_0}^{\theta_m}M(\theta)\mathrm{d}\theta; \tag{31}$$

$$U=\int_{\theta_0}^{\theta_m}\frac{M(\theta)}{g(\theta)}\mathrm{d}\theta+\int_{\theta_l}^{\theta_m}\frac{M(\theta)}{g(\theta)}\mathrm{d}\theta-\frac{e_1+e_3}{2\Delta\varepsilon}In\frac{g(\theta_l)}{g(\theta_0)}; \tag{32}$$

$$\frac{1}{2}l\left\{A-\frac{e_1+e_3}{2g(\theta_0)}P_{s3}|_{z=0}\right\}\sin 2\theta_0=\left\{\int_{\theta_0}^{\theta_m}M(\theta)\mathrm{d}\theta+\int_{\theta_l}^{\theta_m}M(\theta)\mathrm{d}\theta\right\}\left\{\frac{f(\theta_0)}{M(\theta_0)}-\frac{(e_1+e_3)\sin 2\theta_0}{2g(\theta_0)}\right\}, \tag{33}$$

$$\frac{1}{2}l\left\{A+\frac{e_1+e_3}{2g(\theta_l)}P_{s3}|_{z=l}\right\}\sin 2\theta_l = \left\{\int_{\theta_0}^{\theta_m} M(\theta)\mathrm{d}\theta + \int_{\theta_l}^{\theta_m} M(\theta)\mathrm{d}\theta\right\} \left\{\frac{f(\theta_l)}{M(\theta_l)}+\frac{(e_1+e_3)\sin 2\theta_l}{2g(\theta_l)}\right\}. \quad (34)$$

In order to calculated these quantities by a numerical method, we introduce a new variable v to replace θ, as in a previous paper [4],

$$v=\frac{\tan^2\theta}{\tan^2\theta_m}, \quad (35)$$

and let

$$m=\sin^2\theta_m, \quad (36)$$

then

$$\sin\theta=\frac{mv}{1-m+mv}. \quad (37)$$

Let

$$\eta=\frac{k_{33}-k_{11}}{k_{11}},\ \eta'=\frac{\Delta\varepsilon}{\varepsilon_\perp}. \quad (38)$$

The function $M(\theta)$ in equation (26) can be transformed to

$$M(\theta)=\varepsilon_\perp\sqrt{\frac{K_{11}}{\Delta\varepsilon}}\frac{1}{\sqrt{m(1-m)}}\frac{1}{\sqrt{1-v}}M(v), \quad (39)$$

$$M(v)=\sqrt{\frac{[1-m+m(1+\eta)v][1-m+m(1+\eta')v]+e^2m(1-m)v}{1-m+mv}} \quad (40)$$

and the integral in equations (29)–(34) may be re-expressed as

$$\int_{\theta_0}^{\theta_m}\frac{M(\theta)}{g(\theta)}d\theta=\sqrt{\frac{K_{11}(1+\eta' m)}{\Delta\varepsilon}}I_1^{(1)}, \quad (41)$$

$$\int_{\theta_l}^{\theta_m}\frac{M(\theta)}{g(\theta)}d\theta=\sqrt{\frac{K_{11}(1+\eta' m)}{\Delta\varepsilon}}I_1^{(2)}, \quad (42)$$

$$\int_{\theta_0}^{\theta_m}M(\theta)d\theta=\varepsilon_\perp\sqrt{\frac{K_{11}(1+\eta' m)}{\Delta\varepsilon}}I_2^{(1)}, \quad (43)$$

$$\int_{\theta_l}^{\theta_m}M(\theta)d\theta=\varepsilon_\perp\sqrt{\frac{K_{11}(1+\eta' m)}{\Delta\varepsilon}}I_2^{(2)}, \quad (44)$$

where

$$I_1^{(1)}=\int_{v_0}^{1}\frac{1}{2\sqrt{v(1-v)}}\frac{M(v)}{1-m+m(1+\eta')v}dv, \quad (45)$$

$$I_1^{(2)}=\int_{v_l}^{1}\frac{1}{2\sqrt{v(1-v)}}\frac{M(v)}{1-m+m(1+\eta')v}dv, \quad (46)$$

$$I_2^{(1)}=\int_{v_0}^{1}\frac{1}{2\sqrt{v(1-v)}}\frac{M(v)}{1-m+mv}dv, \quad (47)$$

$$I_2^{(2)}=\int_{v_l}^{1}\frac{1}{2\sqrt{v(1-v)}}\frac{M(v)}{1-m+mv}dv. \quad (48)$$

Let

$$I_1=\frac{1}{2}\left(I_1^{(1)}+I_1^{(2)}\right), \quad (49)$$

$$I_2=\frac{1}{2}\left(I_2^{(1)}+I_2^{(2)}\right) \quad (50)$$

and define reduced quantities

$$u=\frac{U}{\left(\pi\sqrt{\frac{k_{11}}{\Delta\varepsilon}}\right)},\ \alpha=\frac{Al}{2k_{11}}\ \text{and}\ e=\frac{e_1+e_3}{\sqrt{k_{11}\varepsilon_\perp}}, \quad (51)$$

which can be termed the reduced voltage, reduced anchoring strength and reduced flexoelectric coefficient, respectively.

Equations (29)–(34) can be now re-expressed as

$$l=\frac{2}{D_3}\sqrt{\frac{k_{11}}{\Delta\varepsilon}}\varepsilon_\perp\sqrt{1+\eta' m}I_2, \quad (52)$$

$$D_3=\frac{2}{l}\sqrt{\frac{k_{11}}{\Delta\varepsilon}}\varepsilon_\perp\sqrt{1+\eta' m}I_2, \quad (53)$$

$$u=\frac{2}{\pi}\sqrt{1+\eta' m}I_1 - \frac{e}{2\pi}In\left(\frac{1-m+mv_0}{1-m+mv_l}\frac{1-m+m(1+\eta')v_l}{1-m+m(1+\eta')v_0}\right), \quad (54)$$

$$d=\frac{1}{D_3}\sqrt{\frac{k_{11}}{\Delta\varepsilon}}\varepsilon_\perp\sqrt{1+\eta' m}I_2^{(1)}=\frac{I_2^{(1)}}{I_2}\frac{l}{2}, \quad (55)$$

$$\alpha\frac{1-m+m(1+\eta')v_0}{1-m+mv_0}-\frac{elP_{s3}|_{z=0}}{2\sqrt{k_{11}\varepsilon_\perp}} = \left\{\sqrt{\frac{1-v_0}{v_0}}M(v_0)-e\sqrt{\frac{1-m+m(1+\eta')v_0}{1-m+mv_0}}\right\}I_2, \quad (56)$$

$$\alpha\frac{1-m+m(1+\eta')v_l}{1-m+mv_l}-\frac{elP_{s3}|_{z=l}}{2\sqrt{k_{11}\varepsilon_\perp}} = \left\{\sqrt{\frac{1-v_l}{v_l}}M(v_l)+e\sqrt{\frac{1-m+m(1+\eta')v_l}{1-m+mv_l}}\right\}I_2. \quad (57)$$

Meanwhile, the solutions of equations (27)–(28) become

$$\frac{z}{l}=\frac{1}{2I_2}\int_{v_0}^{v}\frac{1}{2\sqrt{v(1-v)}}\frac{M(v)}{1-m+mv}\mathrm{d}v \quad \text{for} \quad 0<z<d, \quad (58)$$

$$\frac{z}{l}=1+\frac{1}{2I_2}\int_{v_l}^{v}\frac{1}{2\sqrt{v(1-v)}}\frac{M(v)}{1-m+mv}\mathrm{d}v \quad (59)$$

for $d<z<l$.

4. The threshold voltage and saturation voltage

4.1. *The threshold voltage*

The threshold voltage, U_{th}, corresponds to $m=0$ because $\sin^2\theta_m=0$. From equation (52), we obtain the reduced threshold voltage u_{th} as $u_{th}=\frac{2}{\pi}I_1\Big|_{m=0}=\frac{1}{\pi}\left[\tan^{-1}\sqrt{\frac{1-v_0}{v_0}}+\tan^{-1}\sqrt{\frac{1-v_l}{v_l}}\right]$; u_{th} satisfies

$$\tan\left(\frac{\pi}{2}u_{th}\right)=\frac{\sqrt{\frac{1-v_0}{v_0}}+\sqrt{\frac{1-v_l}{v_l}}}{1-\sqrt{\frac{1-v_0}{v_0}}\sqrt{\frac{1-v_l}{v_l}}}. \quad (60)$$

Using equation (56)–(57), we have

$$\frac{\sqrt{1-v_0}}{\sqrt{v_0}}=\frac{\alpha-\frac{elP_{s3}|_{z=0}}{2\sqrt{k_{11}\varepsilon_\perp}}}{\frac{\pi}{2}u_{th}}+e, \quad (61)$$

$$\frac{\sqrt{1-v_l}}{\sqrt{v_l}}=\frac{\alpha-\frac{elP_{s3}|_{z=l}}{2\sqrt{k_{11}\varepsilon_\perp}}}{\frac{\pi}{2}u_{th}}-e. \quad (62)$$

If $\frac{\mathrm{d}S}{\mathrm{d}z}\Big|_l=-\frac{\mathrm{d}S}{\mathrm{d}z}\Big|_0$ hold approximately by considering the symmetric property, then $P_{s3}|_{z=l}=r_2\frac{\mathrm{d}S}{\mathrm{d}z}\Big|_l=-r_2\frac{\mathrm{d}S}{\mathrm{d}z}\Big|_0=-P_{s3}|_{z=0}$. Defining the reduced strength of surface polarization as

$$p=\frac{elP_{s3}|_{z=0}}{2\sqrt{k_{11}\varepsilon_\perp}}, \quad (63)$$

we have

$$\frac{\sqrt{1-v_0}}{\sqrt{v_0}}=\frac{\alpha-ep}{\frac{\pi}{2}u_{th}}+e, \quad (64)$$

$$\frac{\sqrt{1-v_l}}{\sqrt{v_l}}=\frac{\alpha_2-ep}{\frac{\pi}{2}u_{th}}-e. \quad (65)$$

Equation (60) becomes

$$\cot(\pi u_{th})=\frac{(1+e^2)\pi^2u_{th}^2-4(\alpha-ep)^2}{4\pi u_{th}(\alpha-ep)}. \quad (66)$$

Equation (66) gives the relationship between the reduced threshold voltage, u_{th}, and the reduced strength of surface polarization, p, and shows the influence of surface polarization on NLC cells. Now we estimate the value of p. Equation (63) yields $p=\frac{l}{2}\frac{e_1+e_3}{k_{11}\varepsilon_\perp}r_2\frac{\mathrm{d}S}{\mathrm{d}z}\Big|_0=\frac{l}{2}\frac{(e_1+e_3)^2}{k_{11}\varepsilon_\perp}\frac{1}{\sigma S}\left(-\frac{\mathrm{d}S}{\mathrm{d}z}\Big|_0\right)$, where we use $r_2=-\frac{e_1+e_3}{\sigma S}$ as in Zakharov and Dong [5]. If $|e_1+e_3|\simeq(1\sim10)\times10^{-11}\,\mathrm{C\,m^{-1}}$, $\varepsilon_\perp=(1\sim10)\varepsilon_0$, $\varepsilon_0=8.85\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$, $l=10\,\mu\mathrm{m}$, $k_{11}\sim(1\sim10)\times10^{-12}\,\mathrm{N}$, and taking $S=1$, $\frac{\mathrm{d}S}{\mathrm{d}z}=\frac{\Delta S}{\Delta z}$, $\Delta S=-0.5$ and $\Delta z=10\,\mathrm{nm}$ [5], then $p\simeq l$. The calculated results are shown in figure 1.

4.2. *The saturation voltage*

The saturation voltage corresponds to $m=1$, because $\sin^2\theta_m\approx1$. From equation (54), we obtain the reduced the saturated voltage, u_{sat}, as

$$u_{sat}=\frac{2}{\pi}\sqrt{1+\eta'}I_1\Big|_{m=1} = \frac{1}{\pi}\sqrt{1+\eta}\left[\tanh^{-1}\sqrt{1-v_0}+\tanh^{-1}\sqrt{1-v_l}\right]. \quad (67)$$

The saturation voltage, u_{sat}, satisfies:

$$\tanh\left(\pi\frac{1}{\sqrt{1+\eta}}u_{sat}\right)=\frac{\sqrt{1-v_0}+\sqrt{1-v_l}}{1+\sqrt{1-v_0}\sqrt{1-v_l}} \quad (68)$$

From the boundary conditions of equations (56)–(57),

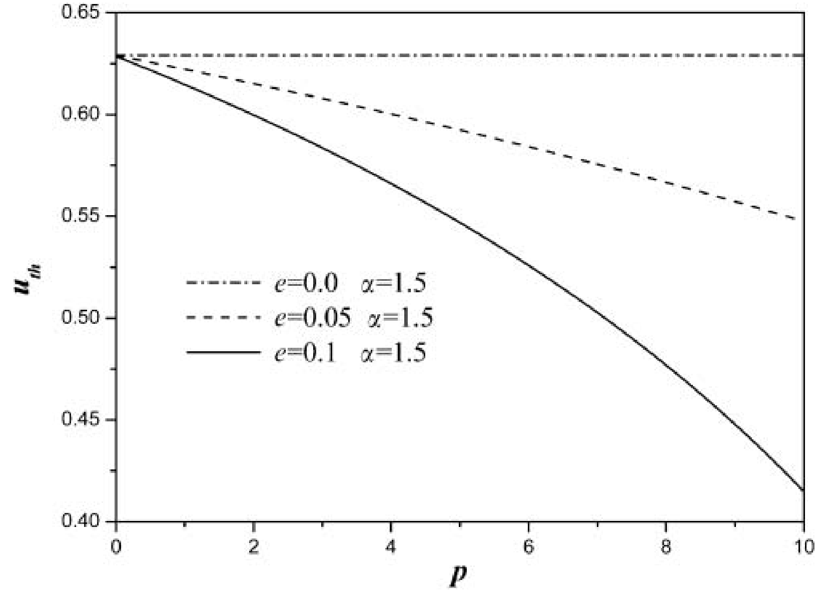

Figure 1. The function of $u_{th}(p)$ for different values of e and α.

we have

$$\sqrt{1-v_0}=\frac{\alpha_1(1+\eta')-\dfrac{elP_{s3}|_{z=0}}{2\sqrt{\varepsilon_\perp k_{11}}}}{\sqrt{1+\eta}\sqrt{1+\eta'}I_2}+\frac{e}{\sqrt{1+\eta}}, \quad (69)$$

$$\sqrt{1-v_l}=\frac{\alpha_2(1+\eta')+\dfrac{elP_{s3}|_{z=l}}{2\sqrt{\varepsilon_\perp k_{11}}}}{\sqrt{1+\eta}\sqrt{1+\eta'}I_2}-\frac{e}{\sqrt{1+\eta}}, \quad (70)$$

where

$$P_{s3}|_{z=0}=r_2\left(1+\frac{r_1}{r_2}\right)\frac{\mathrm{d}S}{\mathrm{d}z}\bigg|_{z=0}, \quad (71)$$

$$P_{s3}|_{z=l}=r_2\left(1+\frac{r_1}{r_2}\right)\frac{\mathrm{d}S}{\mathrm{d}z}\bigg|_{z=l}=-P_{s3}|_{z=0}. \quad (72)$$

Let $r=\frac{r_1}{r_2}$, and using the parameter p provided by equation (63), we obtain

$$\sqrt{1-v_0}=\frac{1}{\sqrt{1+\eta}}\left(\frac{\alpha(1+\eta')-(1+r)ep}{(1+\eta')\dfrac{\pi}{2}u_{sat}}+e\right), \quad (73)$$

$$\sqrt{1-v_l}=\frac{1}{\sqrt{1+\eta}}\left(\frac{\alpha(1+\eta')-(1+r)ep}{(1+\eta')\dfrac{\pi}{2}u_{sat}}-e\right). \quad (74)$$

Substituting equations (73)–(74) into equation (68), we obtain

$$\tanh\left(\pi\frac{1}{\sqrt{1+\eta}}u_{sat}\right)=\frac{4\sqrt{1+\eta}(1+\eta')\pi u_{sat}[\alpha(1+\eta')-(1+r)ep]}{(1+\eta-e^2)(1+\eta')^2\pi^2u_{sat}^2+4[\alpha(1+\eta')-(1+r)ep]^2}. \quad (75)$$

Equation (75) gives the relationship between u_{sat} and reduced anchoring strength, α, reduced flexoelectric coefficient, e, and reduced strength of surface polarization, p. If α and e are known, then the relationship between u_{sat} and p is obtained.

We may take compound MBBA as an example: its elastic constants are $K_{33}=1.25K_{11}$, $K_{11}=5.8\times10^{-12}$ N, $\varepsilon_\perp=5.4$ and $\varepsilon_\parallel=4.7$. Figure 2 shows the curves of u_{sat} versus p, where we take $r=-3$ [9].

5. Discussion

The main result of our theoretical analysis is that the surface polarization of NLC cells is influenced by boundary conditions. This result is consistent with the result obtained by Zakharov and Dong [5], which implied that the surface polarization is mainly influenced by the effective anchoring energy.

In this paper, the flexoelectric polarization is considered but the selective ion absorption is ignored, and this is different from the result of Zakharov and Dong [5]. If selective ion absorption is considered, our work

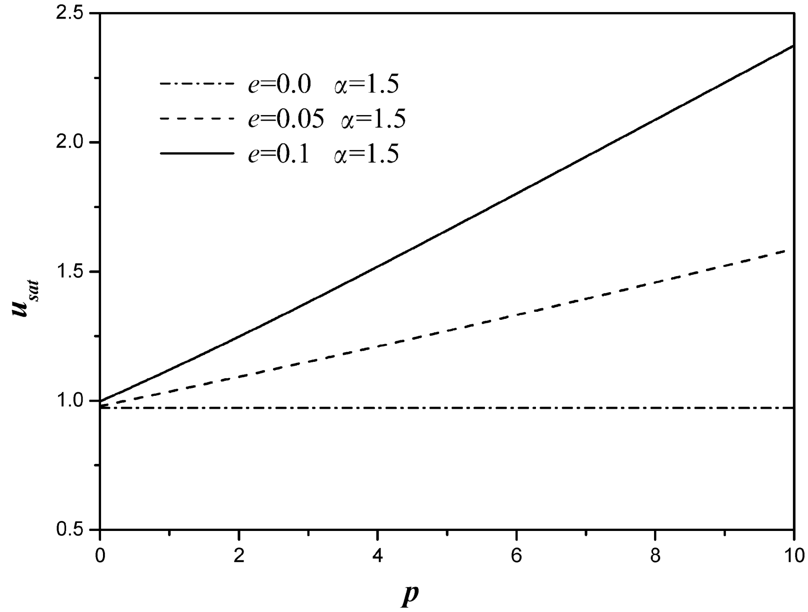

Figure 2. The function of $u_{sat}(p)$ for different values of e and α.

can be generalized. In the theoretical frame, the free energy for the selective ion absorption should be added to the Gibbs free energy, G.

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